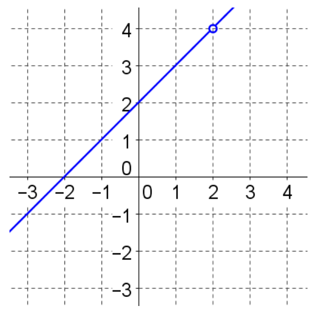
Name:___________________________

SM3 HW3.5: Oblique Asymptotes

If the numerator of a rational function has greater or equal degree than the denominator, then the function is not simplified enough to see everything needed to sketch the function.

We've already seen cases where the denominator divides evenly into the numerator,

$$f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x+2)(x-2)}{x-2} = x + 2; x \neq 2$$

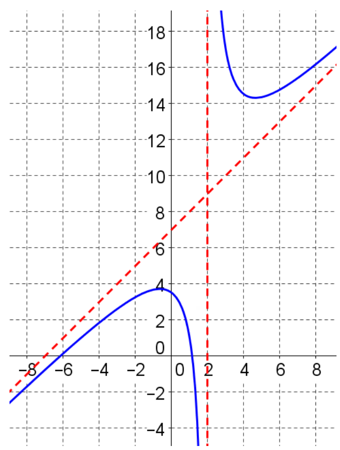


then function simplifies into a polynomial, perhaps with a hole or two.

Today, we'll see cases where the denominator does not divide evenly into the numerator,

$$f(x) = \frac{x^2 + 5x - 7}{x - 2}$$

$$\begin{array}{r|l} & x + 7 + \frac{7}{x-2} \\ \hline x - 2 & x^2 + 5x - 7 \\ & \underline{-(x^2 - 2x)} \\ & \quad 7x - 7 \\ & \quad \underline{-(7x - 14)} \\ & \qquad 7 \end{array}$$



the division has a remainder. The function will still have a vertical asymptote due to the divisor. When the division results in a quotient that has a remainder, the function has an **oblique asymptote**. The portion of the quotient that does not contain the remainder is the oblique asymptote. This item is sketched with a dashed line and represents the end behavior of the function. Then, we just plot a point or two on either side of the vertical asymptote and let the function follow the asymptotes until the ends of the domain.

Example: Sketch $f(x) = \frac{x^3+3x^2-x-2}{x-1}$

$$\begin{array}{r|l} & x^2+4x+3+\dfrac{1}{x-2} \\ x-1 & \overline{x^3+3x^2-x-2} \\ & \underline{-(x^3-x^2)} \\ & \quad 4x^2-x \\ & \quad \underline{-(4x^2-4x)} \\ & \qquad 3x-2 \\ & \qquad \underline{-(3x-3)} \\ & \qquad\quad 1 \end{array}$$

Because the top has more power than the bottom, we're going to have to simplify. While we'd prefer to factor the numerator and check for the chance to cancel, that won't work out with this one, so polynomial long division is the game we'll have to play.

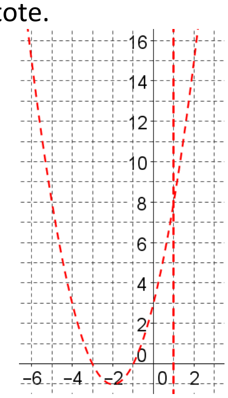
$y = x^2 + 4x + 3$ is the oblique asymptote.

If the remainder is not 0, the portion of the quotient that doesn't contain the $\frac{\text{remainder}}{\text{divisor}}$ is the oblique asymptote.

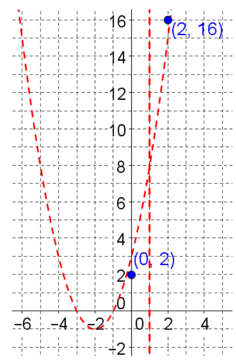
The factor of $x - 2$ in the denominator causes a vertical asymptote of $x = 2$.

Remember, you've got a vertical asymptote too.

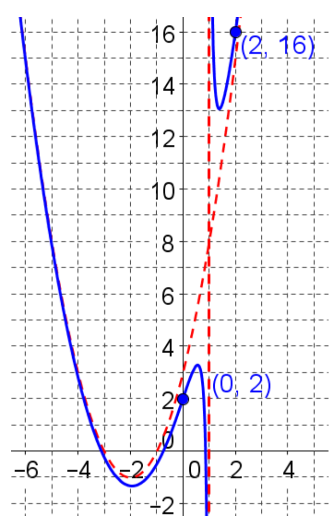
Start your sketch by defining where the function can't exist by drawing your vertical asymptote and oblique asymptote.



Evaluate a couple of points to either side of the vertical asymptote.



Allow your function to follow the asymptotes. Near the vertical asymptote, the function is very far from the oblique asymptote. The farther you get from the vertical asymptote, the closer the function draws to the oblique asymptote.



Perform the polynomial long division or synthetic division (beware of missing terms) to find the oblique asymptote:

1) $f(x) = \dfrac{x^2 + 2x + 1}{x + 3}$

2) $f(x) = \dfrac{2x^3 - 5x - 12}{x^2 - 2}$

3) $f(x) = \dfrac{x^4 - 1}{x - 2}$

Graph the function with any vertical, horizontal, or oblique asymptotes (use dashed lines for asymptotes). Also, plot and label the y-intercept:

4) $f(x) = \dfrac{x^2 + 3x - 5}{x + 2}$

5) $f(x) = \dfrac{x^2 - 1}{x^2 + 3x + 2}$

6) $f(x) = \dfrac{x - 2}{x^2 + x - 6}$

7) $f(x) = \dfrac{x^2 - x - 5}{x + 2}$

8) $f(x) = \dfrac{x^2 - 2x + 1}{x}$

9) $f(x) = \dfrac{x^3 - x^2 - x + 2}{x - 1}$

10) $f(x) = \dfrac{2x^2 - 15x + 27}{x - 5}$

11) $f(x) = \dfrac{x^2 - 2x - 35}{x - 7}$

12) $f(x) = \dfrac{x^3 + x^2 - 3x - 1}{x + 1}$